

# A line-photogrammetric mathematical model for the reconstruction of polyhedral objects

Frank A. van den Heuvel<sup>1</sup>

Delft University of Technology  
Faculty of Civil Engineering and Geosciences  
Thijssseweg 11, 2629JA Delft, The Netherlands

## ABSTRACT

Man-made objects often have a polyhedral shape. For polyhedral objects it is advantageous to use a line-photogrammetric approach, i.e. lines are observed in the images instead of points. A novel line-photogrammetric mathematical model is presented. This model is built from condition equations with image line observations and object parameters in the form of the coordinates of object points and the parameters of object planes. The use of plane parameters significantly simplifies the formulation of geometric constraints. Object line parameters are not included in the model. The duality of the point and plane representation in space is exploited and leads to linear equations for the computation of approximate values. Constraints on the parameters are used to eliminate the rank deficiency and to enforce geometric object constraints. The exterior orientation of the images is assumed to be approximately known. The rotation matrix is parametrised by a unit quaternion.

The main advantages of the presented mathematical model are the use of image lines as observations and the way in which it facilitates the incorporation of all types of geometric object constraints. Furthermore, the model is free of singularities through a combination of over-parametrisation and constraints. The least squares adjustment allows a rigorous assessment of the precision of the computed parameters and allows for statistical testing to detect possible errors in the observations and the constraints. Examples demonstrate the advantages of the proposed mathematical model and show the effects of the introduction of geometric constraints.

**Keywords:** close-range photogrammetry, computer vision, line photogrammetry, geometric constraints, mathematical model, least squares adjustment, polyhedral object reconstruction

## 1. INTRODUCTION

Many man-made objects are constructed using predefined rules that result in regularity of the shape of the object. Planar faces appear frequently, as do symmetries such as parallel and perpendicular object lines and planes. This is definitely the case in the application field of architectural photogrammetry for which the novel mathematical model is primarily designed.

The goal of the research is the adjustment of image line observations and geometric object constraints, in combination with the construction of a polyhedral object description. Advantages of using image lines over points as the basic photogrammetric observations can be found in e.g. [1], [2] and [3]. The image lines are assumed to result from the projection of edges of the object and can be extracted manually or automatically. In both cases the topological relations between image lines and object features are specified manually. The same holds for the object topology. Other premises are the use of a calibrated camera (i.e. the interior orientation including lens distortions is known) and the availability of approximate values for the exterior orientation or pose of each image.

In [4] a solution to this problem was presented that consists of two steps. The first step is the adjustment of the image line observations and the second step is the object reconstruction. The adjustment of image lines does not involve object parameters. The main advantage of this approach is that there is no need for a choice of the parametrisation of the object, nor for approximate values of the object parameters. This makes the approach especially suitable for (sequential) adjustment and statistical testing of observations during interactive modeling when object parameters are not estimable throughout the modeling process. However, a disadvantage is that some geometric object constraints are difficult to formulate; this holds in particular for distance constraints. Another disadvantage lies in the separate step needed for the object reconstruction.

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<sup>1</sup> E-mail: F.A.vandenHeuvel@geo.tudelft.nl

### Reference:

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The approach presented in this paper fully eliminates the disadvantages of the former approach. The drawback is that the new model is less suitable for adjustment during interactive modeling. This is due to the introduction of object point and plane parameters and the need for approximate values of these parameters because of the non-linearity of the model. The plane parameters are introduced to facilitate object constraints like coplanarity of the points, perpendicularity between faces of the object, and distance constraints between parallel planes. The use of object parameters implies a choice for the representation of an object. This is the subject of section 2.

The mathematical model itself including the geometric constraints is discussed in detail in section 3. Approximate values of the point and plane parameters are computed from a possibly redundant set of linear equations (section 4). In section 5 three examples are presented where the mathematical model is applied for the adjustment and reconstruction of parts of a building. The examples differ in the amount of a priori object information that is used.

## 2. GEOMETRIC OBJECT MODELING

In many present day CAD systems there are two approaches to geometric modeling. The first one is the surface or boundary representation (B-rep) of the object and the second one is a solid representation. In the latter case the model is built from a combination of primitives (simple solid shapes like boxes) using Constructive Solid Geometry (CSG). In architectural photogrammetry a surface representation has advantages because very often only parts of a building, like façades, are modeled and a volumetric description with solids is not useful. Furthermore, occlusions of (small) parts of a building prohibit the construction of a solid description without considerable generalization. In our research the object is described by a boundary representation to which geometric object constraints are added. As a first step towards a volumetric description inside and outside of object faces are defined, based on the assumption that the object is viewed from the outside.

The research is restricted to polyhedral objects. Points, lines and planes and their topological relations make up a boundary representation of a polyhedral object. It is most common to only store the 3D coordinates of the points in order to describe the geometry. However, in the case of an object face with a contour that consists of more than three edges (or points) the parametrisation by coordinates is redundant. In such a case constraints are needed to ensure a valid boundary representation. In the object parametrisation adopted here plane parameters are introduced in order to facilitate the formulation of this coplanarity constraint. This constraint then involves the parameters of one point and one plane, while with only point parameters this constraint would involve the parameters of four points. Another reason to introduce plane parameters is the simplification of object constraints, like parallelism, perpendicularity or a distance constraint between parallel planes. These constraints are discussed in section 3.4. Line parameters are not used, mainly because they would imply a considerable burden (see section 4.2 of [4]), although they could simplify the formulation of constraints between lines. If desired, line parameters can be derived from the coordinates of points on the lines. Although no line parameters are included in the model, the presented mathematical model is called a *line-photogrammetric* model because the observations solely consist of image lines. In most of the research on line photogrammetry four or six parameters are used for the object lines ([1], [2], [3], [4], [6], [7]). In [8] parameters for the planes are used, but no coordinates of object points. In [9] the object is described by a combination of polyhedral blocks of which the shape parameters are stored. This approach has many advantages such as the reduction in the number of parameters to recover. However, the constraints used to reduce the number of parameters are absolute (or “hard constraints”, see section 4) and can not be relaxed as in the approach presented in this paper.

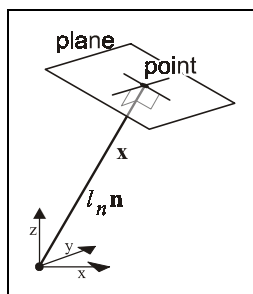


Figure 1: Duality of the point and plane representation

The object parametrisation adopted in the research described here consists of point and plane parameters. Three coordinates of a vector define the position of a point in space. Due to the duality between point and plane representation, a plane can be parametrised by a vector as well, where the orientation of this vector is normal to the plane. The disadvantage of this parametrisation lies in the singularity that occurs when the origin of the coordinate system lies in the plane, because then the

Table 1: Object parametrisation by points and planes

<i>Entity</i>	<i>Orientation</i>	<i>Position</i>	<i>constraints</i>
Point		$\mathbf{x} = (x, y, z)$	
Plane	$\mathbf{n} = (x, y, z)_n$	$l_n$	$\mathbf{R}_i$

orientation of the plane is undefined due to the zero length of the plane vector. To avoid this singularity for each plane an additional parameter is introduced that specifies the distance of the plane to the origin. This over-parametrisation is compensated by a constraint on the length of the normal vector. The object parameters of the mathematical model are summarized in Table 1 and visualised in Figure 1.

### 3. THE MATHEMATICAL MODEL

In this section the novel line-photogrammetric mathematical model is presented. First the observations in the form of image lines are discussed in section 3.1, followed by the parameters in section 3.2. The parameters consist of object parameters (section 2) and exterior orientation parameters. Then the relations between observations and parameters are treated in section 3.3. In section 3.4 the incorporation of geometric object constraints in the model is discussed and in section 3.5 the least squares solution to the model is presented.

#### 3.1 The observations

The mathematical model supports the adjustment of automatically extracted or manually measured straight lines in the image. Assuming a pin hole camera model, an image point observation relates to a spatial direction vector  $\mathbf{d}$  (Figure 2):

$$\mathbf{d} = (x, y, -f) \quad (1)$$

Where  $(x, y)$  is the location of the point in the camera system (corrected for deformations in the image plane and lens distortions) and the focal length  $f$ . The straight line features in the image all have a begin and an end point, possibly on the border of the image. But these points do not have to represent projections of the vertices of the object. This is even unlikely if the lines result from automatic feature extraction. This is the reason why each line feature is represented in the model only through the normal vector  $\mathbf{n}_{ij}$  of the plane. This is the so-called interpretation plane, through line  $j$  and the projection center of image  $i$ . The normal vector can be computed from the direction vectors of begin and end point through their cross product and is rotated to the object coordinate system by multiplication with the rotation matrix of exterior orientation  $\mathbf{R}_i$  (Figure 2):

$$\mathbf{n}_{ij} = \mathbf{R}_i (\mathbf{d}_1 \times \mathbf{d}_2)_j \quad (2)$$

The mathematical model is formulated in such a way that the length of the normal vector  $\mathbf{n}$  does not play a role. Only the image coordinates  $(x, y)$  are treated as observations. The focal length is treated as a constant.

#### 3.2 The parameters

Two types of parameters are distinguished. First the model contains object parameters as discussed in section 2. The second type refers to parameters for exterior orientation of the images.

The object parameters consist of three parameters for each object point and four parameters for a plane. A constraint is needed for each plane as a result of the over-parametrisation (Table 1). The exterior orientation parameters consist of three parameters for the location of the projection center (vector  $\mathbf{r}_i$  for image  $i$ ) and four parameters of a quaternion  $\mathbf{q}$  that defines a rotation matrix  $\mathbf{R}_i$ . These parameters are also called the Euler-Rodrigues symmetric parameters [10]. The major advantage of this parametrisation of the rotation matrix is the absence of singularities (i.e. critical conditions [11]). However, a normalization constraint has to be applied, like with the over-parametrisation used for the planes:

$$|\mathbf{q}| = \sqrt{\mathbf{q}_1^2 + \mathbf{q}_2^2 + \mathbf{q}_3^2 + \mathbf{q}_4^2} = 1 \quad (3)$$

### 3.3 The relations between the observations and the parameters

The main relation used for the mathematical model can be simply stated as *a point is located in the planes to which it relates*. There is only one relation between the observations and the parameters, namely: *an object point is located in the interpretation planes to which it relates* (Figure 2). A similar relation is applied to the object features: *an object point is located in the object planes to which it relates*. The latter relation will be discussed in the next section. The relations between the observations and the parameters (image  $i$ , image line  $j$ , and object point  $k$ ) can be written as:

$$\mathbf{n}_{ij} \cdot \mathbf{x}_k - \mathbf{n}_{ij} \cdot \mathbf{r}_i = 0 \quad (4)$$

It is important to note that in this formulation the length of the normal vector to the interpretation plane  $|\mathbf{n}_{ij}|$  does not play a role and thus the over-parametrisation of the orientation of the interpretation plane does not play a role.

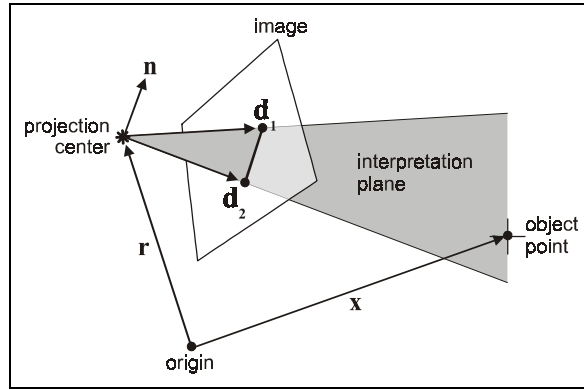


Figure 2: Object point in interpretation plane

### 3.4 The geometric object constraints

The main advantage of the approach presented here is the way in which it facilitates the incorporation of geometric object constraints, also called internal constraints. The constraints are directly imposed on the object parameters, in contrast to the approach in [4], where the object constraints are formulated in terms of the observations (i.e. the normals to the interpretation planes). Two types of geometric constraints can be distinguished. First, the constraints needed for a valid polyhedral object description, i.e. the constraints that force the object points to be in the object plane(s) they belong to. Second, the constraints that result from a priori knowledge on the geometry of the object. These are constraints that affect the shape of the resulting polyhedral object model like parallelism or perpendicularity of object planes. Constraints that affect the definition of the coordinate system, like horizontal or vertical lines or planes, are not dealt with here, but are treated separately in section 3.5.

**Point-point constraint.** There is only one constraint between two points  $i$  and  $j$ , i.e. the distance between them is known:

$$|\mathbf{x}_i - \mathbf{x}_j| = d_{ij} \quad (5)$$

$d_{ij}$ : distance between points  $i$  and  $j$ .

**Line-line constraints.** Apart from a (shortest) distance constraint between two lines, the angle constraint is a constraint with more practical significance. It involves the parameters of four points defining two lines  $i$  and  $j$ :

$$\frac{(\mathbf{x}_{i1} - \mathbf{x}_{i2}) \cdot (\mathbf{x}_{j1} - \mathbf{x}_{j2})}{|\mathbf{x}_{i1} - \mathbf{x}_{i2}| |\mathbf{x}_{j1} - \mathbf{x}_{j2}|} = \cos(a_{ij}) \quad (6)$$

$a_{ij}$ : angle between the lines  $i$  and  $j$ .

**Plane-plane constraints.** The major constraint of this type is the angle constraint between two planes  $i$  and  $j$ :

$$\mathbf{n}_i \cdot \mathbf{n}_j = \cos(a_{ij}) \quad (7)$$

$a_{ij}$ : angle between the plane  $i$  and  $j$ .

Thanks to the (over-)parametrisation chosen for the planes (see Table 1) the distance between two parallel planes  $i$  and  $j$  is simply constrained by:

$$\left| l_{n_i} - l_{n_j} \right| = d_{ij} \quad (8)$$

$d_{ij}$ : distance between planes  $i$  and  $j$ .

It has to be noted that the sign of  $l$  is to be reversed if the normals have an opposite direction. This constraint is linear in contrast to a distance constraint between two points of the object model.

**Point-line constraint.** The only constraint of this type is the shortest distance between a point  $i$  and a line defined by the points  $j$  and  $k$ :

$$\frac{\left| (\mathbf{x}_i - \mathbf{x}_j) \times (\mathbf{x}_k - \mathbf{x}_j) \right|}{\left| \mathbf{x}_k - \mathbf{x}_j \right|} = d_{ijk} \quad (9)$$

$d_{ijk}$ : distance between point  $i$  and line  $j$ - $k$ .

**Point-plane constraint.** This constraint is of the same type as the point-line constraint:

$$\left| \mathbf{x}_i \cdot \mathbf{n}_j - l_{n_j} \right| = d_{ij} \quad (10)$$

$d_{ij}$ : distance between point  $i$  and plane  $j$ .

This constraint is slightly modified to be used as the constraint that forces an object point into the planes it relates to:

$$\mathbf{x}_i \cdot \mathbf{n}_j - l_{n_j} = 0 \quad (11)$$

It has to be noted that the plane parameter  $l_n$  carries a sign that depends on the sign of the normal vector  $\mathbf{n}$ .

**Line-plane constraints.** A distance constraint can be formulated in case of parallelism of line and plane. The angle constraint between a line and a plane is of the form:

$$\frac{(\mathbf{x}_{i1} - \mathbf{x}_{i2}) \cdot \mathbf{n}_j}{\left| \mathbf{x}_{i1} - \mathbf{x}_{i2} \right|} = \sin(a_{ij}) \quad (12)$$

$a_{ij}$ : angle between the line  $i$  and plane  $j$ .

Special cases of some of the geometric constraints need further investigation. These cases occur if the right hand side equals one (equations (6) and (7)) or zero (equations (9) and (12)). Then two equations are needed instead of one.

At the time of writing only the point-plane constraint (10) and the plane-plane angle constraint (7) have been implemented. These constraints are applied in the examples of section 6.

### 3.5 The definition of the coordinate system

The geometric object constraints do not eliminate the rank deficiency of the design matrix that results from the introduction of a coordinate system in combination with parameters for all object points and planes and all exterior orientation parameters. Additional constraints are needed to fix the coordinate system and thereby eliminate the rank deficiency. For this purpose constraints are introduced for a minimum of seven suitable coordinates of object points, although other (combinations of) parameters like the position and/or orientation information of the images could also be used. Over-constraining occurs if more than seven coordinates are available. There are various ways to introduce these constraints in the mathematical model. This is

the subject of the next section. There it is explained why all the constraints are introduced as observation equations. For the coordinate constraints these observation equations have the following form:

$$E\{\mathbf{x}_0\} = \mathbf{I} \mathbf{x}; \mathbf{Q}_{x_0} \quad (13)$$

with:

$E\{\}$	mathematical expectation
$\mathbf{x}_0$	vector of observations (known coordinates of control points)
$\mathbf{x}$	vector of parameters (coordinates of control points)
$\mathbf{I}$	unit matrix
$\mathbf{Q}_{x_0}$	covariance matrix of observations (implemented as a diagonal)

Other constraints that are useful for fixing the coordinate system are the so-called “partial absolute” constraints [12], like horizontal or vertical lines or planes. A vertical line or a horizontal plane constraint fixes two of the seven degrees of freedom of the coordinate system. A horizontal line or a vertical plane fixes one degree of freedom. These constraints on the orientation of the coordinate system involve coordinates of two points (for a line constraint) or the orientation parameters of a plane. If the same partial absolute constraint is enforced on more than one line or plane, the constraints on all but one line or plane can be replaced by parallelism constraints (section 3.4).

#### 4. LEAST SQUARES ADJUSTMENT

In the previous section the observations of the mathematical model, the parameters and their relations have been presented. In this section a least squares solutions to the mathematical model is discussed. The way in which the various types of constraints are enforced in the adjustment leaves room for discussion.

The mathematical model consists of condition equations with observations and parameters. In addition constraints have to be imposed on the parameters. This model can be written as follows (after linearisation, see [13] and [14]):

$$\mathbf{B}^T E\{\mathbf{y}\} = \mathbf{A} \mathbf{x}; \mathbf{0} = \mathbf{C}^T \mathbf{x}; \mathbf{Q}_y \quad (14)$$

with:

$\mathbf{B}^T E\{\mathbf{y}\} = \mathbf{A} \mathbf{x}$	condition equations
$\mathbf{0} = \mathbf{C}^T \mathbf{x}$	constraints on the object parameters
$\mathbf{y}$	vector of (corrections to) the observations (i.e. image line observations)
$\mathbf{x}$	vector of (corrections to) the parameters (i.e. object parameters and exterior orientation parameters)
$\mathbf{A}, \mathbf{B}, \mathbf{C}$	design matrices (partial derivatives)
$\mathbf{Q}_y$	covariance matrix of the observations (implemented as a diagonal matrix)

The condition equations result from linearisation of equation (4) in section 3.3. The constraint equations on the parameters can be of various types; namely the geometric constraints described in section 3.4 or constraints needed for the elimination of rank deficiencies. These rank deficiencies result from over-parametrisation of the object planes (section 2, Table 1) and the rotation matrix (section 3.2, (3)) or result from the use of coordinates (section 3.5, (13)). It is not possible to solve the part of the model with the condition equations first and apply the constraints on the object parameters in a second step. This is caused by the rank deficiency of the system built from the condition equations only ( $\mathbf{A}$  has a rank defect). This is the main reason to transform the model (14) to a model with observation equations only. In [1], [16] and [14] this is called the unified approach. In our approach not all approximate values are treated as observations.

$$E\left\{\begin{matrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{matrix}\right\} = \begin{pmatrix} \mathbf{A} \\ \mathbf{C}^T \end{pmatrix} \mathbf{x}; \mathbf{Q}_{z_1}; \mathbf{Q}_{z_2} \quad \text{or} \quad E\{\mathbf{z}\} = (\mathbf{A}_c) \mathbf{x}; \mathbf{Q}_z \quad (15)$$

with:

$\mathbf{z}_1 = \mathbf{B}^T \mathbf{y}$	vector of (corrections to) the derived observations
$\mathbf{Q}_{z_1} = \mathbf{B}^T \mathbf{Q}_y \mathbf{B}$	covariance matrix of the derived observations
$\mathbf{z}_2 = \mathbf{0}$	vector of (corrections to) pseudo or constraint observations
$\mathbf{Q}_{z_2}$	covariance matrix of the pseudo observations (diagonal)

The model consists of two parts that are solved simultaneously. Due to the transformation of constraints into pseudo observations the complete system ( $\mathbf{A}_C$ ) is of full rank. In the first part the condition equations lead to observation equations with derived observations ( $\mathbf{z}_1$ ) of which the (full) covariance matrix ( $\mathbf{Q}_{z_1}$ ) is derived by propagation. In the second part pseudo observations are introduced of which the variances are chosen to be very small compared to the variances of the original and derived observations. This holds for the variances of the “constraint observations” that are used to eliminate the rank deficiency resulting from the over-parametrisation. For other constraints, like the constraints for the coordinates of control points, a realistic value for the variance of the pseudo observations can be chosen. The same applies to the geometric object constraints. The variance adopted for the pseudo observation reflects the uncertainty in the constraint. In practice, it is difficult to find realistic values for these variances because they will differ considerably from one application to the other. The variance determines the weight of the constraint in the adjustment. A very large weight will result in an object model that fully complies with the constraint, a realistic weight will allow discrepancies between adjusted model and constraint. These different weightings are sometimes referred to as hard and soft constraints (“weak forms” in [12]).

The least squares solution to the model (15) is well-known:

$$\hat{\mathbf{x}} = (\mathbf{A}_C^T \mathbf{Q}_z^{-1} \mathbf{A}_C)^{-1} \mathbf{A}_C^T \mathbf{Q}_z^{-1} \mathbf{z} \quad (16)$$

The residuals of the derived and pseudo observations are computed as follows:

$$\hat{\mathbf{e}}_z = \mathbf{z} - \mathbf{A}_C \hat{\mathbf{x}} \quad (17)$$

The residuals of the original image line observations are computed from the first part of the model (15):

$$\hat{\mathbf{e}}_y = \mathbf{Q}_y \mathbf{B} \mathbf{Q}_{z_1}^{-1} (\mathbf{z}_1 - \mathbf{A} \hat{\mathbf{x}}) \quad (18)$$

It has to be noted that  $\mathbf{B}^T$  has to be of full rank for an invertible  $\mathbf{Q}_{z_1}$  (and  $\mathbf{Q}_z$ ). This can only be the case if no more than two condition equations per image line are used. Therefore, it is not possible to relate more than two object points to one interpretation plane. Object constraints have to be used to force more than two points onto an object line.

Because we deal with a non-linear model in both the parameters and the observations, a so-called mixed model [13], the solution is computed in an iterative procedure in which parameters and observations are updated. This implies that equation (18) is evaluated in each step of the iteration process. This is not the case in a conventional (non-mixed) non-linear model with observation equations.

## 5. APPROXIMATE VALUES

Initial values are needed for all parameters in the model because the equations presented in the previous section are non-linear. Two types of parameters have to be distinguished. These are the object parameters and the parameters of exterior orientation or pose parameters of the images. The computation of approximate values for exterior orientation is discussed first.

There are several ways to obtain approximate values for the exterior orientation parameter of the images. Direct methods for space resection based on a priori information on object coordinates are available [17]. In [18] a method is proposed that makes use of the parallelism of coplanar object lines. A direct solution has been derived for which only the projection of (parts of) the edges of a parallelogram (or rectangle) in object space is needed. If the same parallelogram is visible in two images, relative orientation between them can be determined. If a specific object coordinate system is to be used a similarity transformation has to be applied.

With the exterior orientation parameters (approximately) known, the object point parameters are computed using equation (4). This equation is linear if exterior orientation parameters ( $\mathbf{R}$  and  $\mathbf{r}$ ) and observations (represented by the normal vector to the interpretation plane  $\mathbf{n}$ ) are treated as constants. Equations of this type are gathered and a least squares adjustment is performed to compute the coordinates of the object point ( $\mathbf{x}$ ). This adjustment per object point allows for an assessment of the estimability of the coordinates of the point. In case of an overdetermined system of equations, statistical testing can be applied to detect errors in the observations or errors in the approximate values for the exterior orientation parameters. A minimum configuration of three interpretation planes for an object point is depicted in Figure 3. Poor intersection angles between interpretation planes are detected in the approximate value computation of each object point.

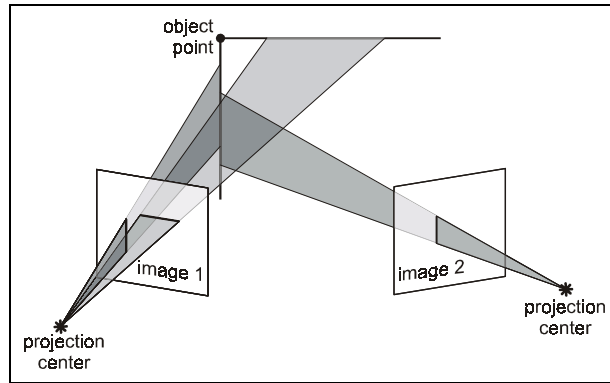


Figure 3: Minimum configuration

The plane parameters are computed after the computation of object point coordinates. Now equation (11) is applied with  $\mathbf{x}$  and  $l_n$  as constants. Then these equations are gathered for each plane and the three parameters of  $\mathbf{n}$  are solved with  $l_n$  set to one. Normalization of  $\mathbf{n}$  results in  $l_n$  being the length of the original vector  $\mathbf{n}$ . Equation (11) is also linear and, in fact, identical to the rewritten equation (4) used for the point coordinate computation. Points and planes are treated in the same way in the approximate value computation, making use of the point-plane duality discussed in section 2. It has to be noted that the computation of approximate plane parameters is not singularity-free.

With the availability of the plane parameters, additional equations are available for the computation of the coordinates of the points that are specified to be in that plane. Point and plane parameters are computed in an alternating way until no new point or plane parameters are computed. With this approach it is possible to compute approximate values for object points that are visible in only one image, or not in any image, if there are at least three object planes intersecting at that point. The parameters of these planes then have to be previously computed from the coordinates at least three other points in the plane. The result is that, due to the point-plane constraints (section 3.4), it can occur that parts of the model visible in less than two images can be reconstructed. This is demonstrated by the examples described in the next section.

## 6. EXAMPLES

The mathematical model presented in this paper is applied for modeling the entrance part of our department building. Three images were taken with a digital camera (Kodak DCS420 with a 20 mm lens) of which the interior orientation (including lens distortion) is known. Lines were measured manually by measuring a start and end point of each straight line. These points were chosen in such a way that the line in the image is as long as possible and the edge showed good contrast. The three images and the image line observations are visualised in Figure 4. Note that there is no correspondence between points in different images. The standard deviation of the observations was chosen to be constant at 1 pixel for all tests. When lines are extracted automatically the stochastic model can be chosen in accordance with the precision estimation from the line extraction method. A stochastic model in which the variance of the coordinates of the end points depends on the length of the measured line is presented in [19].

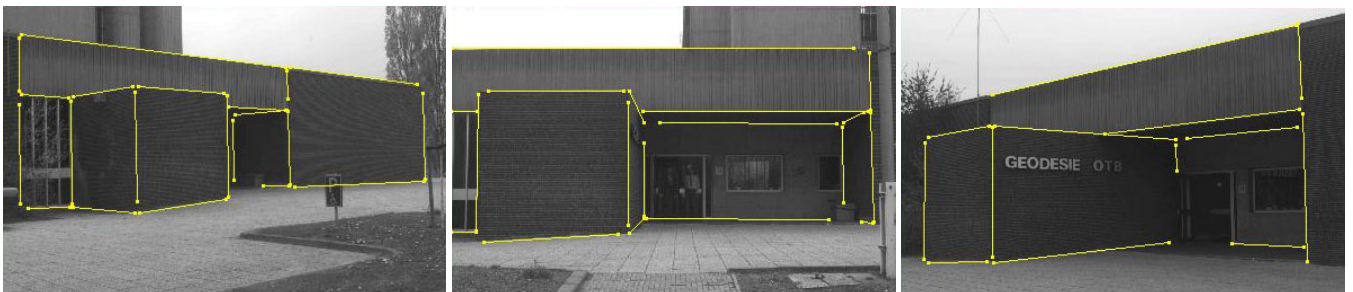


Figure 4: Image line measurements

Advantages of line-photogrammetric measurement over point measurement are illustrated by the details in Figure 5. In the top left picture three object points at the corners of three faces are visible (in the center of the three clusters of end points) while in

the top right picture only one of the three points could have been measured had point measurements been used. The other two corner points are not visible because one point is off image and the other point is occluded. With line measurements a part of the edge between the two missing points can be measured and in this case that line is crucial for the reconstruction. In the two pictures at the bottom of Figure 5 occlusion of a corner point by a box that is not modeled does not prohibit the reconstruction of the object point because a sufficient number of image lines relating to this point can be measured.

Apart from the image line measurements the object topology was specified i.e. the faces, the edges and the vertices of the object and their relations were defined. Furthermore, each image line was related to two vertices of the object. This is the specification of what is called the image-object topology. Approximate values for exterior orientation parameters were obtained with the use of parallelism information on edges of the object [18]. Seven approximate object coordinates were fixed to define the coordinate system of the model (section 3.5).

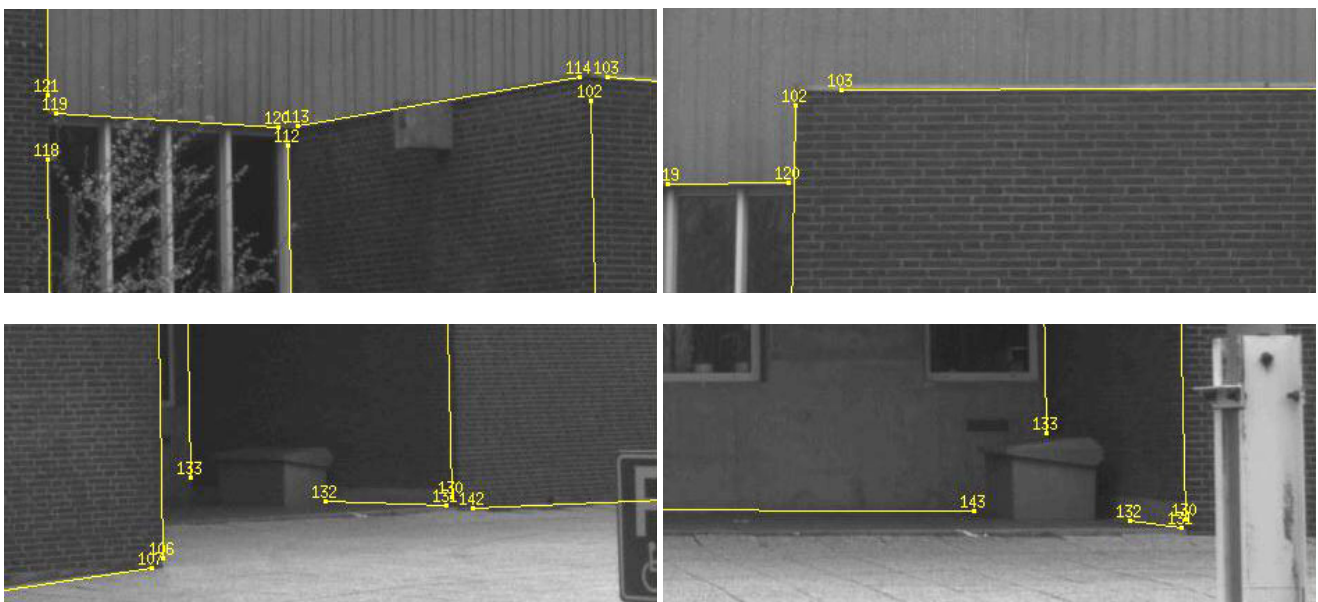


Figure 5: Details of left and middle image

Three adjustments have been performed, all with the same set of line measurements and the same stochastic model. The adjustments only differ in the amount of object information supplied. The main characteristics of the adjustments are summarized in Table 2. The three resulting texture-mapped models can be accessed through the Internet at [www.geo.tudelft.nl/frs/architec/triple.html](http://www.geo.tudelft.nl/frs/architec/triple.html).

In the first adjustment all faces are treated independently, i.e. there are no object constraints apart of the constraints that force a corner point into the plane of a face. The resulting model is pictured in Figure 6. In each point of the model an ellipsoid is positioned in order to visualize the precision of the coordinates. The scale of all ellipsoids is a factor 10 larger than the scale of the model. Furthermore the ellipsoids of the position of the three projection centers is visible in the top views. The precision of the coordinates is dependent on the choice for the definition of the coordinate system. Here, the three coordinates of the two points on the left of the brick wall in the front are fixed using equation (13) with (very) large weights. The seventh coordinate that is fixed in this way is located at the top right of the same wall. The size and shape of the ellipsoids depend on this choice, but it does not affect the precision of the shape of the model. For instance, the precision of distances computed from the coordinates is not affected by the choice of the coordinate system.

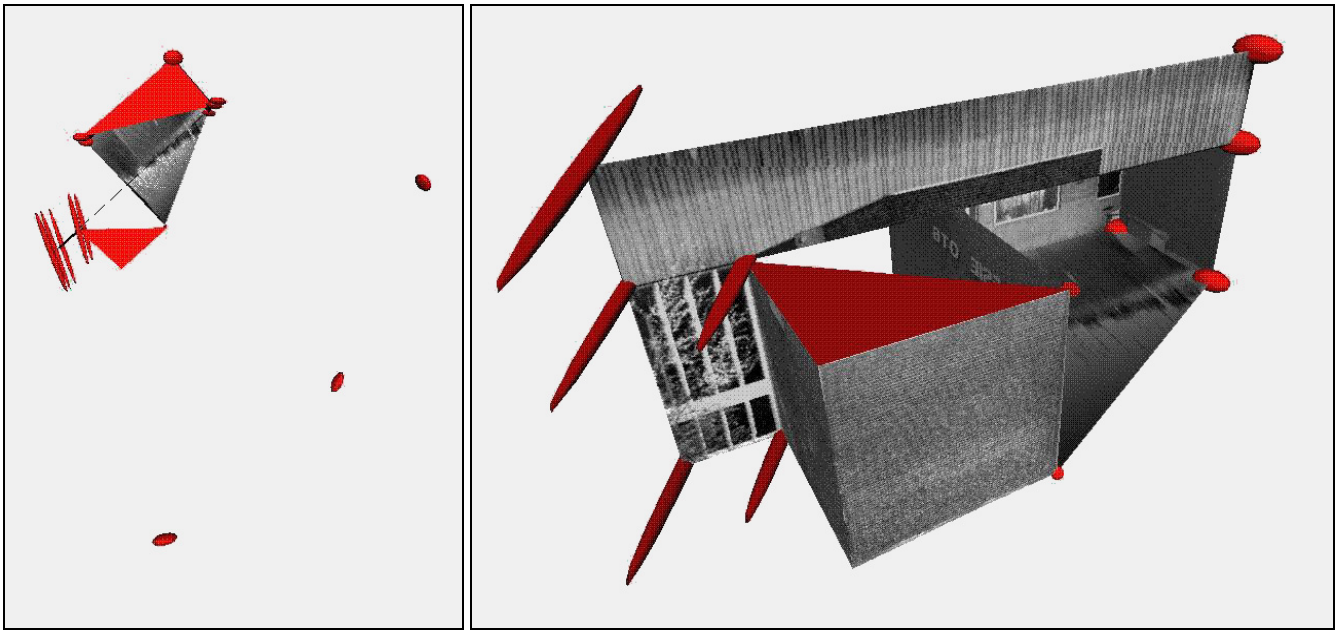


Figure 6: Top (left) and oblique view of the first model (see [www.geo.tudelft.nl/frs/architec/triple.html](http://www.geo.tudelft.nl/frs/architec/triple.html))

Two auxiliary triangular horizontal faces have been defined. They do not play a role in the first adjustment. Note the five elongated ellipsoids on the left side of the model. This is due to small intersection angles between interpretation planes associated with approximately horizontal lines in the images. For better intersection angles the images should have been taken from different height levels. These five ellipsoids point towards the projection center of the leftmost image because only this image contains two line measurements for each of these points (see Figure 4). Information on the precision is summarized in Table 3. The number of iterations is determined by the criterion for the maximum correction to the parameters (set to  $10^{-6}$ ).

Table 2: Number of parameters and redundancy

<i>Adjustment</i>	<i>Points</i>	<i>Planes (faces)</i>	<i>Parameters</i>	<i>Equations (<math>\mathbf{A}_C</math>)</i>	<i>Equations (<math>\mathbf{B}^T</math>)</i>	<i>Redundancy</i>
1	16	8 (10)	101	149	98	48
2	18	8 (11)	107	160	102	53
3	18	8 (11)	107	171	102	64

In the second adjustment additional object information is incorporated in the form of coplanarity constraints. Three vertical faces are assumed to be in one plane and the same holds for the two triangular horizontal roof faces. As a result, the number of plane parameters reduces by eight (two planes less, but one additional face). Because of these additional constraints the coordinates of a new face become estimable. Furthermore, the precision of the coordinates improves, which is reflected by smaller ellipsoids (see Figure 7 and Table 3). The number of planes is the same for all three adjustments (Table 2) because in the first adjustment there are no plane parameters introduced for two horizontal triangular faces. There is no need for those parameters because these planes do not involve constraints. In the second adjustment there is one set of plane parameters for these two faces and one set of parameters for the three faces that make up the front of the building.

Table 3: Results of the adjustments

<i>Adjustment</i>	<i>Iterations</i>	<i>Estimated standard deviation (pixel)</i>	<i>Largest standard deviation of object coordinate (percentage of object size)</i>
1	4	0.67	1.7%
2	4	0.74	0.41%
3	5	1.37	0.28%

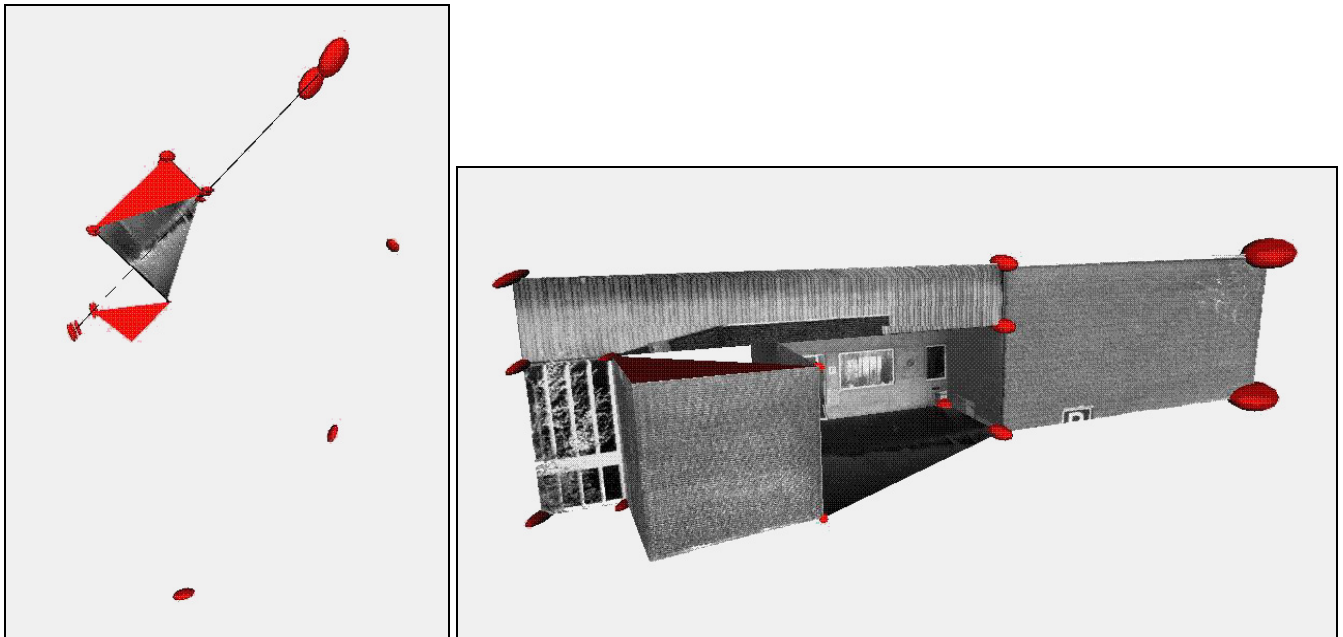


Figure 7: Top and oblique view of second model

In the third adjustment a new type of geometric object constraint is introduced. This is the angle constraint between two planes. In total eleven perpendicularity constraints were applied. All vertical planes were set perpendicular to the horizontal roof plane (six constraints) and each vertical plane was set perpendicular to its neighboring vertical plane (five constraints). From Figure 8 and Table 3 it is clear that the formal precision is improved by the additional object information. However, the estimated standard deviation increases considerably. This raises the question whether there are model errors present. Is the stochastic model too optimistic, is the camera model correct (interior orientation), and are all the object constraints valid in their absolute form? Statistical testing of individual constraints and image line observations can provide an answer, but is not implemented yet. Furthermore, realistic weighting of the constraints and its effect on precision is to be investigated. The same holds for the reliability aspects of the presented mathematical model.

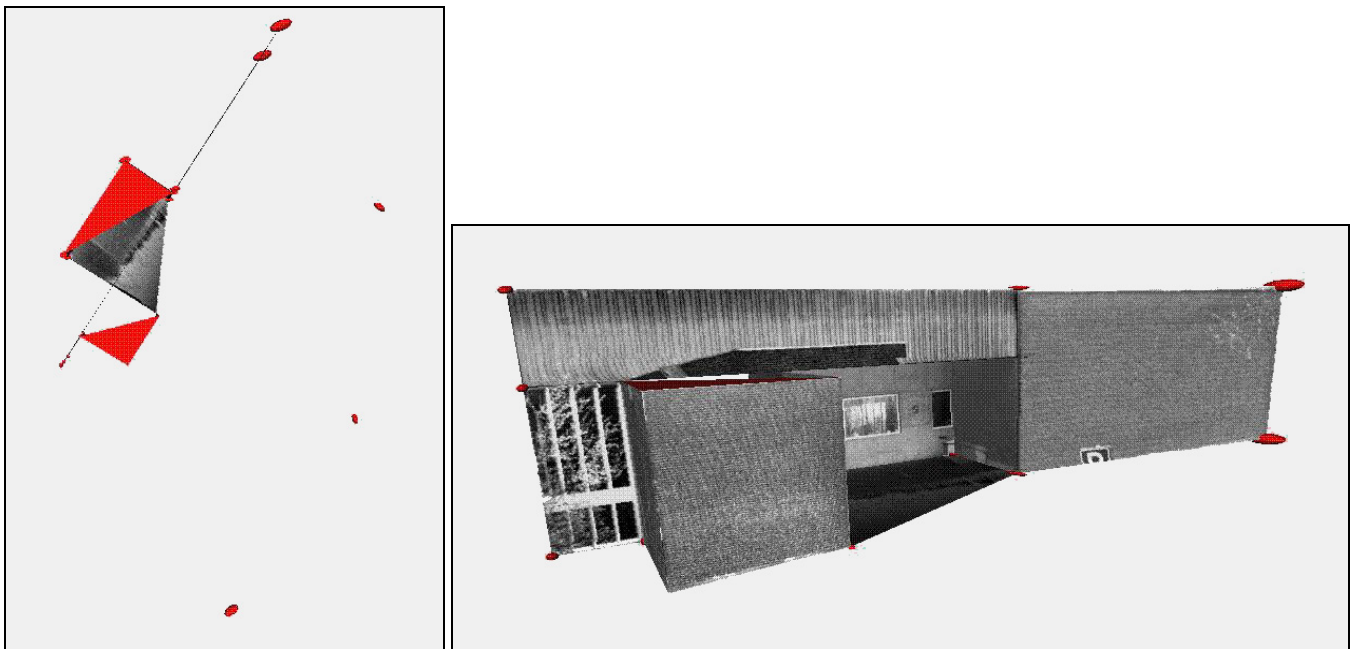


Figure 8: Top and oblique view of third model

## 7. CONCLUSIONS

A new mathematical model for the reconstruction of polyhedral objects using line photogrammetry is presented. The main difference from existing models is the use of condition equations with observations and parameters. Apart from parameters for the exterior orientation of the images, parameters for object points and planes are incorporated in the model. The plane parameters facilitate the formulation of geometric object constraints, such as coplanarity, and angle or distance constraints between planes of the object. Over-parametrisation in combination with constraints for the rotation matrix and the planes result in a mathematical model free of singularities. Approximate values of all object parameters are computed from, possibly redundant, sets of linear equations.

The examples demonstrate the advantages of the approach. Due to the use of line photogrammetry occluded object points can be reconstructed. Parts of the building visible in only one image can be modeled due to the introduction of coplanarity constraints. Furthermore, the formal precision improves significantly with the introduction of constraints like perpendicularity between object planes. The validation of the object constraints in combination with their weighting is to be investigated.

## REFERENCES

1. H. Zielinski, *Object reconstruction with digital line photogrammetry*, Doctoral thesis Department of Geodesy and Photogrammetry, Royal Institute of Technology, Stockholm, 1993
2. P. Patias, E. Petsa, A. Streilein, *Digital Line Photogrammetry*, IGP Bericht nr. 252, Eidg. Technische Hochschule, ISBN 3-906513-73-4, Zürich, 1995
3. E.M. Mikhail, K. Weerawong "Exploitation of linear features in surveying and photogrammetry", *Journal of Surveying Engineering*, Vol. 123, No. 1, pp. 32-47, 1997
4. F.A. van den Heuvel, G. Vosselman, "Efficient 3D-modeling of buildings using a priori geometric object information", *Videometrics V*, Sabry F. El-Hakim (ed.), SPIE Vol. 3174, pp.38-49
5. E. Guelch, "Line photogrammetry: a tool for precise localization of 3D points and lines in automated object reconstruction", *Integrating Photogrammetric Techniques with Scene Analysis and Machine Vision II*, SPIE Vol. 2486, pp.2-12, Orlando, 1995
6. D. Mulawa, *Estimation and photogrammetric treatment of linear features*, Doctoral thesis, Purdue University, 1989
7. R. Schwermann, *Geradengestützte Bildorientierung in der Nahbereichsphotogrammetrie*, Doctoral thesis, Geodätischen Instituts der Rheinisch-Westfälische Technischen Hochschule Aachen, ISSN 0515-0574, Aachen, 1995
8. S. Weik, O. Grau, "Recovering 3-D object geometry using a generic constraint description", *International Archives of Photogrammetry and Remote Sensing*, Vol.31, Part B3, pp.593-598, Vienna, 1996
9. P.E. Debevec, C.J. Taylor, J. Malik, "Modeling and rendering architecture from photographs: a hybrid geometry- and image-based approach", *Computer Graphics proceedings*, Annual Conference Series, pp.11-20, New Orleans, 1996
10. M.D. Shuster, "A Survey of Attitude Representations", *Journal of the Astronautical Sciences*, Vol. 41, No. 4, pp. 439-517, 1993
11. T.-Y. Shih, "The Duality and Critical Condition in the Formulation and Decomposition of a Rotation Matrix", *Photogrammetric Engineering and Remote Sensing*, Vol. 56, No. 8, pp. 1173-1179, 1990
12. H.F. Barakat, K. Weerawong, E.M. Mikhail, "Feature-based Photogrammetric and Invariance Techniques for Object Reconstruction", *International Archives of Photogrammetry and Remote Sensing*, Vol.31, Part B3, pp. 535-541, Vienna, 1996
13. P.J.G. Teunissen, *Adjustment Theory*, Lecture notes Faculty of Geodetic Engineering, Delft University of Technology, Delft, 1994
14. E.M. Mikhail, *Observations and Least Squares*, IEP-A Dun-Donnelley Publisher, New York, 1976
15. A. Streilein, U. Hirschberg, "Integration of digital photogrammetry and CAAD: constraint-based modelling and semi-automatic measurement", *CAAD Futures '95 International Conference*, Singapore, 1995
16. C. McGlone, "Bundle adjustment with geometric constraints for hypothesis evaluation", *International Archives of Photogrammetry and Remote Sensing*, Vol.31, Part B3, pp. 529-534, Vienna, 1996
17. R.M. Haralick, C.-N. Lee, K. Ottenberg, M. Noelle, "Review and analysis of solutions of the three point perspective pose estimation problem", *Int. Journal of Computer Vision*, Vol. 13, No. 3, pp.331-356, 1994
18. F.A. van den Heuvel, "Exterior Orientation using Coplanar Parallel Lines", *10<sup>th</sup> Scandinavian Conference on Image Analysis*, Lappeenranta (Finland), pp.71-78, 1997
19. F.A. van den Heuvel, "Vanishing Point Detection for Architectural Photogrammetry", *International Archives of Photogrammetry and Remote Sensing*, Vol.32, Part 5, pp. 652-659, Hakodate, 1998