ESTIMATION OF INTERIOR ORIENTATION PARAMETERS FROM CONSTRAINTS ON LINE MEASUREMENTS IN A SINGLE IMAGE

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ABSTRACT

In this paper a method is presented for the estimation of interior orientation parameters and lens distortion using measurements of lines in a single image and constraints derived from a priori information on the orientation of the related object lines. The interior orientation parameters consist of the location of the principle point and the focal length. Radial lens distortion is modeled with one parameter. The straight image lines are extracted automatically using a line-growing algorithm. The constraints on the orientation of the object lines are parallelism and perpendicularity constraints. These constraints are specified automatically using a vanishing point detection algorithm. The two-step procedure starts with the least-squares estimation of the lens distortion using only parallelism constraints. In this step, the image line observations are checked for blunders and their precision is assessed. In the second step, the principal point and the focal length are estimated using adjusted observations from the first step and constraints for perpendicularity in addition to the parallelism constraints. The formal precision of the parameters results from propagation of variances. The precision turned out to be in the order of 0.1 mm for the tests performed with wide-angle imagery (focal length 9.2 mm, sensor size 8.6x6.9 mm^2).
The procedure is applied to several images of a building and the results of the parameter estimation are evaluated. The quality of the estimated interior orientation parameters depends strongly on the orientation of the image relative to the object and the focal length of the camera. Furthermore, the availability and distribution of object lines and constraints plays an important role.
This automatic procedure for the estimation of lens distortion and interior orientation parameters is well suited to application to historic imagery taken with an unknown camera, or for imagery taken with a camera with an unstable interior orientation. This holds true for wide-angle, three-point perspective (i.e. no object edges are parallel to the image plane) imagery of objects showing straight lines that can be assumed parallel and perpendicular.

1. INTRODUCTION

It is common practice in photogrammetry to estimate the interior orientation parameters of a camera from multiple images of a point field (Fryer, 1996). The camera is assumed stable during a longer period. When self-calibration is applied, the camera is assumed stable during image acquisition only, and coordinates of the points need not be known a priori. These standard procedures cannot be used for a camera with unstable interior orientation or when geometric information is to be extracted from (single) images taken with a camera of which the interior orientation is unknown. The method presented here uses constraints on lines in the image that result from a priori information on the geometry of the related lines in object space. The major advantage of this line-photogrammetric method over the use of point measurements, is the possible application of automatic line-extraction procedures. Furthermore, the constraints are inferred automatically by a vanishing point detection procedure. No other information on the position or orientation of the object lines is required.
A method that uses geometric object constraints for camera calibration (including exterior orientation) from point measurements in multiple images, is presented in (Youcai and Haralick, 1999). Graphical and analytical methods for the estimation of interior orientation parameters from vanishing points in a single image can be found in (Williamson and Brill, 1990).

Reference:
F.A. van den Heuvel, 1999. Estimation of interior orientation parameters from constraints on line measurements in a single image. International Archives of Photogrammetry and Remote Sensing, Vol. 32, Part 5W11, pp. 81-88

2. ESTIMATION OF INTERIOR ORIENTATION PARAMETERS

The procedure for estimation of the interior orientation using a single image consists of three steps:

1. Extraction of straight lines from the image.
2. Specification of parallelism and perpendicularity between the corresponding lines in object space.
3. Estimation of the interior orientation parameters and lens distortion using these constraints.

2.1 Straight line extraction

Due to lens distortion, straight lines in object space appear as slightly curved lines in the image. Straight lines are extracted using a line-growing algorithm. Any algorithm for straight-line extraction will break up long curved image lines into smaller sections. The object edges that relate to these sections are parallel and, without lens distortion, the image lines would intersect in a single point, the so-called vanishing point. Therefore, lens distortion is to be removed for vanishing point detection. In this paper, the estimation of the lens distortion is discussed, using the same vanishing point intersection constraint (van den Heuvel, 1998a).

2.2 The geometric constraints

Two types of constraints are applied, namely parallelism and perpendicularity constraints. The constraints can be specified manually or automatically using a vanishing point detection algorithm (van den Heuvel, 1998a). In principle, all known angles between (groups of parallel) lines of the object can be used, but only parallelism and perpendicularity, as the most common angles in man-made objects, are implemented.
Both constraints are formulated using the normals to the interpretation planes of the image lines involved. The image lines are represented by the image coordinates of the end points (figure 1). The image coordinates (*x, y*) are related to a direction vector in object space ($\mathbf{d}$ in the coordinate system of the camera) (Fryer, 1996):

$$\mathbf{d} = \begin{pmatrix} x - x_o - k_1(x - x_o) r^2 \\ y - y_o - k_1(y - y_o) r^2 \\ -f \end{pmatrix} \qquad (1)$$

with:

f : effective focal length
x_o , y_o : coordinates of the principle point
k_1 : radial lens distortion parameter
$r^2 = (x - x_o)^2 + (y - y_o)^2$

Lens distortion is often modeled more extensively. Although the model presented here can be extended, the major part of the lens distortion is eliminated by applying parameter k_1 only.
The normal to the interpretation plane of the image line *i*, with end points 1 and 2, is found using:

$$\mathbf{n}^i = \mathbf{d}_1 \times \mathbf{d}_2 \qquad (2)$$

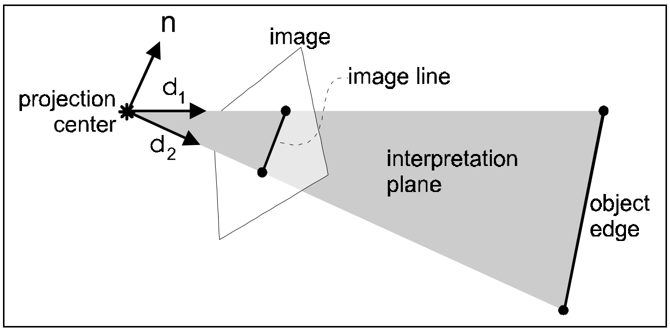

Figure 1: The interpretation plane

2.2.1 The parallelism constraint

The parallelism constraint can be written as the determinant of the matrix built from the normal vectors to the three interpretation planes of lines *i*, *j* and *k* (van den Heuvel and Vosselman, 1997):

$$\left[\mathbf{n}^i, \mathbf{n}^j, \mathbf{n}^k\right] = \det(\mathbf{n}^i, \mathbf{n}^j, \mathbf{n}^k) = 0 \qquad (3)$$

As can be seen from equations (1) and (2), equation (3) contains parameters (x_o, y_o, f, k_1) and observations (image coordinates *x, y*). In order to set up a least-squares adjustment this equation is linearized with respect to both the parameters and the observations. This leads to a so-called mixed model (Teunissen, 1994):

$$\mathbf{B}^{\mathrm{T}} E\{\mathbf{y}\} = \mathbf{A}\,\mathbf{x} \;;\; \mathbf{Q}_{\mathbf{y}} \qquad (4)$$

with:

$\mathbf{y}$	vector of (corrections to) the observations
$\mathbf{x}$	vector of (corrections to) the parameters
$\mathbf{A}, \mathbf{B}$	design matrices (partial derivatives)
$\mathbf{Q}_{\mathbf{y}}$	covariance matrix of the observations

The covariance matrix $\mathbf{Q}_{\mathbf{y}}$ of the image points is implemented as a diagonal matrix. Several options are available for the stochastic model:

- If lines are extracted using an edge detection approach, the variances of the coordinates of the end points are assumed to decrease linearly with the length of the line, in pixels.

- For lines that are extracted manually by measurement of their end points, a constant variance is assumed for all end point coordinates.

Line orientation does not play a role in the stochastic model as only precision information perpendicular to the line affects the precision of the interpretation plane normals.

2.2.2 The perpendicularity constraint

The perpendicularity constraint involves the interpretation plane normals of four lines, consisting of two perpendicular sets (1 and 2) of two lines that are parallel in object space. This constraint can be written as:

$$\left(\mathbf{n}_1^i \times \mathbf{n}_1^j\right) \cdot \left(\mathbf{n}_2^k \times \mathbf{n}_2^l\right) = 0 \qquad (5)$$

In this equation, the vector that results from the cross product has the same spatial orientation as the related object edges. The result of the dot product relates to the angle between orientations 1 and 2.

Like the parallelism constraint, this constraint is linearized with respect to the parameters of interior orientation and the observations. The system of condition equations (4) is extended with linearized perpendicularity constraints.

2.2.3 Selection of constraints

The least-squares solution to the system (4) is discussed in section 2.3. In order to be able to compute the solution, all constraints have to be independent. This is achieved in the following way. First, for each object orientation the two image lines are selected that best define the object orientation (the longest vector that results from a cross product in equation 5). Let us call these two lines the *base* lines. Then for each of the *n*-2 remaining lines of that orientation, parallelism constraints are set up (*n* is the number of lines in a cluster of parallel lines). Each constraint involves three lines, two of which are the base lines. In this way, *n*-2 independent constraints are defined for each object orientation.

Second, a maximum of three perpendicularity constraints is set up for the three major object orientations (*X, Y, Z*). These constraints are formulated using the base lines of each object orientation that have been used for the parallelism constraints as well. The three independent constraints define perpendicularity between three combinations of two object axes (*XY, YZ,* and *ZX*).

The selection of parallelism and perpendicularity constraints presented here, leads to an independent set of condition equations. Other selections of independent constraints are possible and will lead to the same results, in principle. However, numerically unfavorable selections have to be avoided.

In this paper, we concentrate on estimation of interior orientation parameters from a single image. However, if several images taken with the same camera are available, the lines and constraints of all images can be used to build the system of condition equations (4). Of course, the interior orientation of the camera is assumed identical for all the images.

2.3 Parameter estimation

The system of condition equations (4) is transformed into a standard system of observations equations by the introduction of the so-called derived observations ($\mathbf{z}$):

$$E\{\mathbf{z}\} = \mathbf{A}\,\mathbf{x}\,;\ \mathbf{Q}_\mathbf{z} \qquad (6)$$

with:

$\mathbf{z} = \mathbf{B}^\mathrm{T}\mathbf{y}$	vector of (corrections to) the derived observations
$\mathbf{Q}_\mathbf{z} = \mathbf{B}^\mathrm{T}\mathbf{Q}_\mathbf{y}\mathbf{B}$	covariance matrix of the derived observations

The covariance matrix $\mathbf{Q}_\mathbf{z}$ (treated as a full matrix) of the derived observations results from propagation of the diagonal covariance matrix of the observations $\mathbf{Q}_\mathbf{y}$. The least-squares solution to this system is well known:

$$\hat{\mathbf{x}} = (\mathbf{A}^\mathrm{T}\,\mathbf{Q}_\mathbf{z}^{-1}\,\mathbf{A}\ )^{-1}\,\mathbf{A}^\mathrm{T}\,\mathbf{Q}_\mathbf{z}^{-1}\,\mathbf{z} \qquad (7)$$

The computation of the residuals of the original observations is explained in (van den Heuvel, 1999).

Since the mathematical model is non-linear in both the parameters and the observations, the iteration process needs special attention. In principle, convergence has to be obtained by iterating in the parameters only. After that, iteration in both the parameters and the observations leads to a set of parameters and observations that perfectly fit the model. For this iteration in parameters and observations, only one iteration step was needed.

Experiments showed that no convergence of the iteration in the parameters could be obtained when the interior orientation parameters were estimated. Convergence was only obtained when the lens distortion parameter was estimated, and the principal point and focal length were not. The reason for the absence of convergence in the parameter iteration lies in the sensitivity of the solution to the coefficients in the design matrix ($\mathbf{A}$). These coefficients depend on the observations and are imperfect due to noise in the observations (and possibly blunders). Using adjusted observations, the iteration in the parameters converges well.

3. ESTIMATION IN TWO STEPS

In this section, separating the estimation of the parameters in two steps extends the procedure described in the previous section:

1. Estimation of radial lens distortion.
2. Estimation of interior orientation.

This separation in two steps is also made in (van den Eelaart and Hendriks, 1999). Furthermore, the first step of the procedure can be classified as an "on-the-job plumb-line calibration" (Fryer,1996).

The mathematical model is identical to the one described in section 2. However, in the first step of the procedure only parallelism constraints are applied. In the second step, both types of constraints are used to estimate the parameters using adjusted observations from the first step. In other words, the (adjusted) image lines used in the second step all intersect at their vanishing points. In principle, only the perpendicularity constraints are needed in the second step. The parallelism constraints are added for the assessment of the formal precision of the parameters. The solution is identical to the one without parallelism constraints, and identical to the direct solution from three perpendicular sets of two parallel lines (called *base* lines in the previous section) published in (Kraus, 1996).

A few remarks to the two-step procedure:

- The solution of the first step does not depend on the focal length, so an arbitrary initial value can be selected.
- However, the solution of the first step depends on the initial location of the principle point. In the examples described in the next section, the center of the image format is chosen as the point of symmetry. Iteration in the initial position of the principle point is possible, but was not examined.
- The residuals of the observations are not affected in the second step. The reason lies in the one-to-one relation between the location of the three (adjusted) vanishing points resulting from the first step, and the three parameters of interior orientation. This relation is established by the three independent constraints for perpendicularity. Therefore, statistical testing of the observations is performed in the first step of the procedure, and not in the second step.

4. EXAMPLES

The method for estimation of interior orientation parameters from a single image is applied to the calibration of an Olympus C1400 digital camera. The imaging sensor contains 1280 by 1024 pixels (pixel size 6.7 µm). The camera has a zoom lens of which the two extreme settings were used, i.e. 9.2 mm and 28 mm nominal focal length. In the following, these two settings are called the wide-angle and narrow-angle settings, respectively. The lens system was focussed at infinity. At both settings, three images of a Delft University building were taken from street level. The camera settings were identical for the three images. The first image was taken roughly perpendicular to the façade. The second one is an oblique image of the same façade and the third image is the only one that contains all three major object orientations.

Figure 2: First wide-angle image with extracted lines (top) and corrected for radial lens distortion (bottom).

4.1 Wide-angle imagery

The first wide-angle image (9.2 mm lens), overlaid with the automatically extracted image lines, is shown in figure 2.
The number of extracted lines varies with the parameter settings for the line-growing algorithm. The settings used resulted in 230 lines. The vanishing point detection resulted in two sets of lines, 55 vertical and 145 horizontal lines. It has to be noted that the nominal values for the interior orientation have been used for the vanishing point detection. This implies that the principle point is located at the center of the image format and the focal length is set to its nominal value (9.2 mm). If this information is not available, a minimum number of lines (three perpendicular sets of two parallel lines) has to be measured manually. The a priori standard deviation used for the vanishing point detection was chosen to be relatively high to ensure that the lens distortion did not hinder the clustering:

$$\sigma_o = \frac{0.1\,\text{mm}}{\sqrt{\text{line length (pixel)}}}$$

In all the tests for the estimation of the interior orientation parameters, the a priori standard deviation of the end point coordinates (σ_o) is fixed at 5 μm or 0.75 pixel. The estimation of the lens distortion parameter using the original observations (the first step of the procedure) showed this value to be realistic for most experiments (see the test values ($\frac{\hat{\sigma}^2}{\sigma_0^2}$) in Table 1). Maximum residuals of end point coordinates are sometimes large (around 4 times the a priori standard deviation). This can be due to the limited model for the lens distortion, or imperfections in the line clustering of the vanishing point detection procedure.

In the first step of the procedure, the parameter for radial lens distortion was estimated using only parallelism constraints. The results are summarized in Table 1. Figure 2 shows the image corrected for radial lens distortion.
In the second step, the interior orientation parameters could not be estimated. The location of the principle point cannot be estimated because only two object orientations are available. The focal length cannot be estimated because the image is taken perpendicular to the façade, and therefore the angle between the two orientations in object space (reconstructed from the image line observations) is not affected by the focal length.

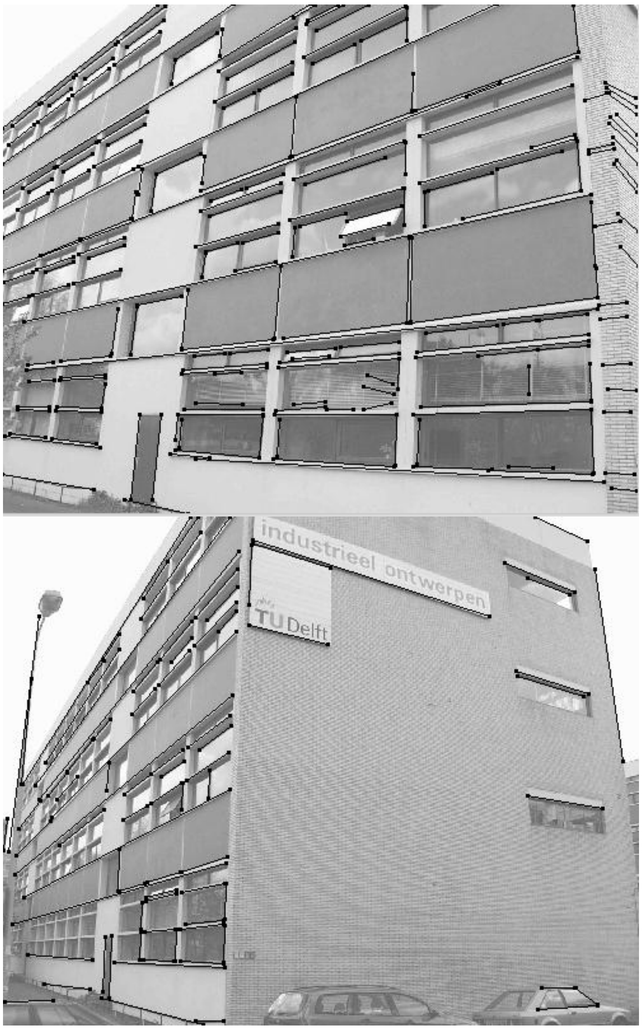

Figure 3: Oblique wide-angle imagery with extracted lines; façade (top) and three-point perspective (bottom).

The second and the third image were taken with the same camera settings as the first (Figure 3). The second image is an oblique one of a single façade, and does not allow estimation of the principle point. However, the focal length can be estimated if the principle point is assumed to be in the center of the image. The third image has a so-called three-point perspective (i.e. no object edges parallel to the image plane) and thus all interior orientation parameters can be estimated (Williamson and Brill, 1990). Note that due to the lack of windows in one of the façades, the number of lines and their distribution over the image is not optimal. The results of the parameter estimation are summarized in Table 1.

Wide-angle	Number of constraints par. + per.	k_1 (10^{-3} mm^{-2})	x_o (mm)	y_o (mm)	f (mm)	$\hat{\sigma}^2 / \sigma_0^2$	Maximum residual (µm)
1. Perpendicular	198 (+1)	-1.627	-	-	-	0.68	21.5
		(0.061)					
2. Oblique façade	163 + 1	-1.823	-	-	9.212	1.33	20.2
		(0.078)			(0.115)		
3. Oblique	69 + 3	-1.685	-0.162	-0.154	9.250	0.87	8.8
		(0.135)	(0.165)	(0.045)	(0.044)		

Table 1 : Parameter estimation results for the wide-angle images (standard deviation between brackets).

Narrow-angle	Number of constraints par. + per.	k_1 (10^{-3} mm^{-2})	x_o (mm)	y_o (mm)	f (mm)	$\hat{\sigma}^2 / \sigma_0^2$	Maximum residual (µm)
1. Perpendicular	247 (+1)	0.953	-	-	-	0.38	9.9
		(0.059)					
2. Oblique façade	326 + 1	1.127	-	-	26.984	2.08	31.1
		(0.081)			(2.93)		
3. Oblique	194 + 3	1.261	2.59	0.868	26.388	0.82	24.8
		(0.145)	(1.17)	(0.502)	(0.561)		

Table 2 : Parameter estimation results for the narrow-angle images (standard deviation between brackets).

4.2 Narrow-angle imagery

The three narrow-angle images (28 mm nominal focal length) show the same perspective as the three wide-angle images. However, the narrow-angle images have two related disadvantages in comparison to the wide-angle images. First, the longer focal length leads to a degradation of the precision of the location of the vanishing points, due to a more unfavorable intersection of image lines. Second, the longer focal length is also the reason that the images are taken almost parallel to the vertical object orientation. For the third image, the perspective is very close to a two-point perspective (i.e. one object orientation is parallel to the image plane), for which the principle point cannot be estimated (Williamson and Brill, 1990). This explains why the precision of x_o is significantly lower than the precision of y_o for both the wide-angle (close to a two-point perspective) and the narrow-angle image.

Due to the two disadvantages mentioned, the method presented in this paper is not suited for the estimation of the interior orientation parameters of narrow-angle images. As can be concluded from Table 2, this holds especially for the estimation of the principle point (in *y*-direction the standard deviation amounts 0.5 mm or 75 pixels). However, for reconstruction purposes with this type of imagery, errors in the location of the principle point will have only minor effects on the results of the reconstruction. Therefore, the center of the image format can usually be taken to represent the principle point with sufficient precision.

The estimation of the lens distortion does not depend on the focal length. It can be estimated for both wide- and narrow-angle images with similar precision.

Figure 4: The narrow-angle images (28 mm lens).

5. CONCLUSIONS

A method for the estimation of lens distortion and interior orientation parameters from a single image, using constraints on line measurements, has been presented. The constraints on the image line observations are derived from geometric constraints on the related lines in object space. The object line constraints are parallelism and perpendicularity between the three major orientations of the object edges. These assumptions are valid for many man-made objects and they occur very frequently in buildings. Therefore, this method is suitable for historic images of buildings, taken with an unknown camera. However, the orientation of the image relative to the object is crucial for the estimation of the interior orientation parameters. For the estimation of lens distortion the image orientation does not play a role.

The characteristics of the method can be summarized as follows:

- Least-squares estimation of lens distortion and interior orientation parameters using line measurements in a single image.
- The only object information required, is the parallelism and perpendicularity of object edges.
- The precision of the parameters is assessed by rigorous error propagation.
- When an approximate value for the focal length is available, the method is fully automatic, using a straight-line extraction and a vanishing point detection algorithm. If this is not the case, a few manual measurements are required.
- The method is only applicable to images of objects with parallel and perpendicular edges.
- For the estimation of the interior orientation parameters, the orientation of the image relative to the object plays a crucial role. A so-called three-point perspective is required.
- The precision of the estimated interior orientation parameters is generally higher when the focal length is shorter.

Camera calibration is not a goal in itself. Calibration is a necessity for accurate 3D measurement. The application of the presented calibration procedure has not been tested in combination with 3D reconstruction, possibly from the same (single) image using the same constraints (van den Heuvel, 1998b). The use of the object constraints in the 3D reconstruction will reduce the effects of errors in the parameters of the camera model. However, it will not eliminate them. The use of the calibration procedure in combination with 3D reconstruction is a topic for future work.

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